

Math Tools and Strategies for Differentiating Instruction and
Increasing Student Engagement

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Math Tools and Strategies for Differentiating Instruction and Increasing Engagement

Our Thoughtful questions:

- *Why do some students succeed in mathematics while others do not? Is it a matter of skill or will?*
- *How can we use research-based teaching tools and strategies to address the styles of all learners so they succeed in mathematics?*

Our workshop is based on the following assumptions:

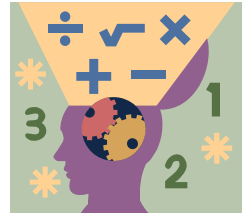
- *What teachers do and the instructional decisions they make have a significant impact on what students learn and how they learn to think.*
- *Different students approach mathematics using different learning styles and need different things from their teachers to achieve in mathematics.*
- *Style-based mathematics instruction is more than a way to invite a greater number of students into the teaching and learning process; it is, plain and simple, good math—balanced, rigorous, and diverse.*

In this workshop, you will learn:

- *The characteristics of the four basic mathematical learning styles (Mastery, Understanding, Self-Expressive, and Interpersonal) and how to assess both your own mathematical teaching style and your students' mathematical learning styles.*
- *How to use a variety of mathematical teaching tools and strategies to differentiate instruction and increase student engagement.*

The Human Element of Mathematics

Describe a personal incident in your life using as many mathematical terms as you can (in the box below). Then meet with a neighbor to share and compare your stories. Keep count of how many mathematical terms are used.



Mathematical Story

What Is Mathematical Literacy?

Mastery of procedural and conceptual knowledge. <hr/>	A language to communicate ideas and solve real-world problems. <hr/>
Understanding of logical reasoning to explain and prove a solution. <hr/>	Application of strategies to formulate and solve problems. <hr/>

What percentage of your classroom practice would you estimate you spend in each of these areas? (Write your percentage on the line in each box above).

How does your classroom practice compare with the NAEP data?

NAEP data shows that proficiency in these four areas has developed unevenly. In many classrooms, students are able to mimic rules and procedures demonstrated by their teacher: however, students often acquire these skills with little depth of understanding or the ability to use them to solve complex problems (Kowley & Wearing 2000).

Mastery Learners

In general, Mastery learners want to work and do well in school. When they withdraw from learning, many of their problems can be addressed cognitively according to four basic principles:

- Mastery students are motivated by clarity, competence, and success. Motivation starts with clear expectations. Set explicit and measurable goals for both academic achievement and behavior. The better Mastery students know the criteria for evaluating their performance, the more they will work to meet them.
- The skills necessary for success hold whether the skill is fairly straightforward (e.g., studying for a test on factoring) or it's a more abstract and complex skill, such as making inferences, establishing a thesis for an essay, or developing a plan for solving a word problem. The more explicitly the teacher models the skill, the stronger Mastery students will perform.
- Mastery students need lots of practice on key skills and central concepts. It is not necessary to reduce the thinking, reading, writing, or problem-solving in their work; it is only necessary to provide more practice and better feedback on their performance.
- When it comes to learning complex content, Mastery students need organizational tools. They frequently fool us because they are so good at following directions, but their tendency to focus on details makes it difficult for them to see and use the higher-level concepts necessary to understand abstract content. The use of visual organizers, as well as effective modeling and practice of study skills, can provide an effective boost to Mastery learners.

Possible Solutions for Mathematics

- Focus work in mathematics around a math log where students do fewer, but more complex problem-solving assignments. Insist that students illustrate and prove the reasoning they use.
- Emphasize the role of diagramming in interpreting and solving problems in mathematics.
- Use samples of high, medium, and low student problem-solving, and explanations of reasoning to provide a touchstone to help Mastery learners assess their progress.
- Model reasoning and explaining processes frequently.
- Make sure that a high percentage of student work involves word problems.
- Provide visual organizers at the beginning of units to show the central concepts and kinds of problems that will be addressed.
- Use manipulatives, but remember that for Mastery learners two of the best tools are money and graph paper.

Understanding Learners

In general, Understanding learners like to be engaged in critical thinking and academic learning. When they withdraw from learning, many of their problems can be addressed cognitively according to four basic principles:

- Increase the intellectual content of the curriculum and the complexity of the thinking tasks assigned.
- Provide clear reasons for routine work and a system that permits Understanding learners to measure and assess their own progress in these areas.
- Where the curriculum calls for exploration of personal experiences or cooperative group work, model explicitly how this work is done but also permit discussion of why it is important.
- Emphasize the role of reflection in deep learning. Model and practice with students how to become aware of the thinking and attention processes they are using in solving problems or collecting information.

Possible Solutions for Mathematics

- Focus work in mathematics on solving a small number of complex problems rather than a larger number of simpler exercises.
- Explicitly model for students how to observe and take notes on their problem-solving processes *while* they are doing math.
- Explicitly model and practice how to use notes taken during the problem-solving process to build specific explanations.
- Provide a visual organizer at the beginning of a unit to show not just the central concepts and kinds of problems that will be addressed, but the intriguing questions that will be explored as well (e.g., "When is a fraction superior to a percentage and vice versa?").
- Make sure Understanding students can compute easily and well, but emphasize the use of mental math rather than either routine algorithms or calculators.

Self-Expressive Learners

In general, Self-Expressive learners need opportunities for choice, to express their creativity, and to pursue project work that stimulates their imaginations. Whenever learning becomes rote or they have few or no opportunities to pursue their interests, Self-Expressive students may lose their motivation. When they withdraw from learning, many of their problems can be addressed cognitively according to four basic principles:

- Increase the imaginative stimulation in the content by focusing on large and engaging ideas, investigating curious and mysterious objects, going on field trips, telling stories, and working on imaginative projects.
- Provide more sustained time for reading, writing, problem-solving, and research.
- Ensure that there are frequent opportunities for coaching and conversation.
- Explicitly model and practice all routine and organizational skills.

Possible Solutions for Mathematics

- Focus mathematics learning on problem-solving (first), writing and illustrating mathematical ideas (second), and computational precision (third).
- Regularly emphasize the relationship between art and mathematics.
- Wherever possible, replace worksheets on computation with practice in mental computation where students solve problems in their heads and then discuss and compare strategies.
- Explicitly model and practice computational algorithms and the use of formulas only after students have taken considerable time to explore mathematical ideas.
- Explicitly teach students how to use their natural tendency to form images in their mind's eye to create diagrams for the problems they are solving.

Interpersonal Learners

In general, Interpersonal learners want to "experience" learning, and forge social connections during the process. As learning becomes increasingly abstract and seems to have little to do with human feelings and personal experiences, these learners often lose motivation. When they withdraw from learning, many of their problems can be addressed cognitively according to four basic principles:

- Connect content and tasks as closely as possible to students' experiences and real-world contexts, especially those connected to the students' own communities.
- Increase preparation for, and implementation of, learning partners (student pairs) and cooperative learning groups.
- Build more opportunities for discussion and the sharing of personal opinions and values into the learning process.
- Use explicit modeling, practice, feedback, and organizational strategies to develop students' capacities for handling abstract concepts and complex tasks.

Possible Solutions for Mathematics

- Continually provide opportunities for students to solve complex math problems through conversation and collaboration in small groups and learning partnerships.
- Provide opportunities for students to teach their new mathematical ideas to others.
- Model and practice a variety of ways of representing mathematical ideas and procedures visually and verbally.
- Use "prove-it" sessions to practice mental computation, and then explain the *how* and the *why* of the strategies students used to perform these mental computations.
- Use home-based and community-based mathematics projects to explore how mathematics can be used as a real-life learning tool.

Teaching *With* Style

Teaching *with* style is..

Implementing a variety of instructional teaching tools, strategies, and activities to differentiate instruction in order to support and challenge each student's learning profile.

Four Styles of Teaching

S T E P S	T R U S T
P R O B E	I M A G E

Five Ways to Use Math Tools

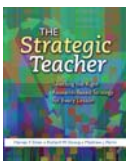
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Math Notes

Purpose: Math Notes helps students build notemaking skills as they examine key components of word problems from multiple angles and work to develop thoughtful solutions.


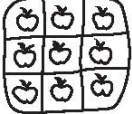


Steps:

1. Model the notemaking process with an example and a blank Math Notes organizer. Make sure students “hear” your thinking while you break the problem down into:
 - The Facts Identify the facts of the problem and determine what is important, what isn’t important, and what is missing.
 - The Question Determine the main question that needs to be answered as well as any hidden questions that are important to solving the problem.
 - The Diagram Sketch a visual representation of the problem.
 - The Steps Decide what steps need to be taken to solve the problem.
2. Have students practice solving problems on their own—they should collect their Math Notes work in a problem-solving notebook.
3. Move students to independent use of Math Notes. When they encounter new problems, have students review their notebooks and look for effective problem-solving models to guide them.



For more about this tool, see pages 212-213 in *The Strategic Teacher: Selecting the Right Research-Based Strategy for Every Lesson*.

Example:

<p>The Problem: There are three 5th grade classes going on a field trip. Two classes have 22 students, and one class has 21 students. The elementary school wants to purchase a juice box for each student going on the field trip. Juice boxes are sold in nine-packs or four-packs. How many juice boxes does the school need? What is the fewest number of packs the school needs to buy so that every student gets a juice box and the school has the least possible number of leftover juice boxes?</p>	
<p style="text-align: center;">The Facts</p> <p>What are the facts?</p> <ul style="list-style-type: none"> • Large packs contain nine juice boxes. • Small packs contain four juice boxes. <p>What is missing?</p> <ul style="list-style-type: none"> • Number of students in the 5th grade classes. • Number of juice boxes the school should buy. 	<p style="text-align: center;">The Steps</p> <p>What steps can we take to solve the problem?</p> <ul style="list-style-type: none"> • Find out how many students are going on the trip. That's how many juice boxes we need. • See how many sets of nine I can get out of the number of students. • I know I can't get any more sets of nine when I have a number lower than nine left. • When I get to a number lower than nine, see if it is better to buy a nine-pack or a four-pack.
<p style="text-align: center;">The Question</p> <p>What question needs to be answered?</p> <ul style="list-style-type: none"> • How many nine-packs and four-packs does the school need to buy? <p>Are there any hidden questions that need to be answered?</p> <ul style="list-style-type: none"> • How many people are going on the trip? • Is there more than one right answer? 	<p style="text-align: center;">The Diagram</p> <p>How can we represent the problem visually?</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p>= 1</p> </div> <div style="text-align: center;">  <p>= 9</p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 10px;"> <div style="text-align: center;">  <p>= 4</p> </div> <div style="text-align: center;">  <p>= 65</p> </div> </div>

Source: Silver, H.F., Strong, R.W., & Perini, M.J. (2007). *The Strategic Teacher: Selecting the Right Research-Based Strategy for Every Lesson*. (p.213)

Math Notes Organizer

<p>The Facts</p> <p><i>What are the facts?</i></p> <p><i>What is missing?</i></p>	<p>The Steps</p> <p><i>What steps can we take to solve the problem?</i></p>
<p>The Question</p> <p><i>What question needs to be answered?</i></p> <p><i>Are there any hidden questions that need to be answered?</i></p>	<p>The Diagram</p> <p><i>How can we represent the problem visually?</i></p>
<p>The Solution</p>	

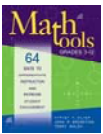
The Canoe Problem

Nineteen campers are hiking through Acadia National Park when they come to a river. The river is moving too rapidly for the campers to swim across. The campers have one canoe, which fits three people. On each trip across the river, one of the three canoe riders **must** be an adult. There is only one adult among the nineteen campers. How many trips across the river will be needed to get all of the children to the other side of the river?



Source: Silver, H.F., Thomas, E., & Perini, M.J. (2003) *Math Learning Style Inventory*. (p.2)

Mastery | Fist Lists and Spiders



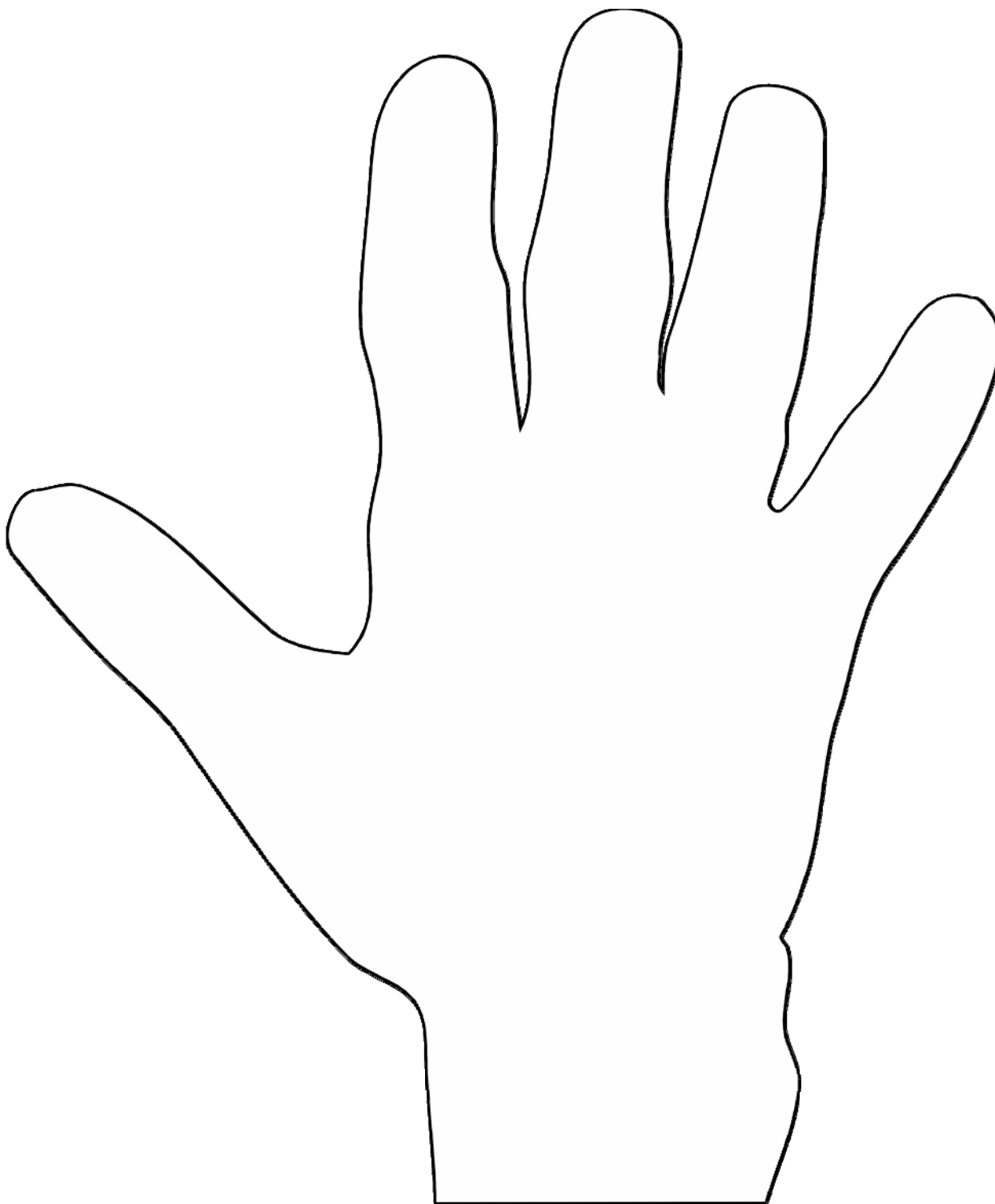
For more about this tool, see pages 29-32 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: Fist Lists and Spiders help students build and master critical vocabulary by mapping out the connections between mathematical ideas. When creating a Fist list or Spider, students identify and then visually connect important ideas, words, attributes, characteristics, or procedures that are strongly related to the mathematical concept at hand.

Steps:

1. Identify a mathematical concept or critical term for students to consider.
2. Provide students with a Hand or Spider Organizer, or allow students to create their own.
3. Have students write the mathematical concept or term being discussed in the center.
4. Allow students time to think about the focus of their maps and to generate ideas. Have students write down their five or eight best ideas, one in each digit of their Fist List or on each leg of their Spider.
5. Have students share and discuss their maps with a partner or within a small group.
6. Encourage students to share their Fist Lists or Spiders with the entire class and explain the connections that they made between their key ideas and the central mathematical concept or term.

Fist List



Understanding | Always-Sometimes-Never (ASN)



For more about this tool, see pages 66-69 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: Always-Sometimes-Never (ASN) is a reasoning activity that focuses students thinking around the important, and often subtle, facts and details associated with mathematical concepts. Students are asked to consider statements containing mathematical information and determine if what is stated is always, sometimes, or never true.

Steps:

1. Provide students with a list of statements about a recently discussed or familiar mathematical concept or topic.
2. Allow students enough time to read and consider all of the statements carefully.
3. Have students think about each of the statements and decide whether each is *always true*, *sometimes true*, or *never true*.
4. Make sure that students explain the reasoning behind their choices.

Examples:

Arithmetic: Addition and Subtraction

1. The sum of two 3-digit numbers is a 3-digit number (sometimes)
2. The sum of two even numbers is an odd number (never)
3. The difference of two odd numbers is an even number. (always).
4. The sum of additive inverses is zero. (always)

Statistics: Mean, Median, Mode

1. A list of numbers has a mean. (always)
2. A list of numbers has a median. (always)
3. A list of numbers has a mode. (sometimes)
4. The mean of a set of numbers is one of the numbers of that set. (sometimes)

Trigonometry: Graph Analysis

1. The graph of a trigonometric function is periodic. (always)
2. Doubling the amplitude of a trigonometric function doubles the period of the function. (never)
3. The graph of a cosecant function has an infinite number of asymptotes. (always)

Always-Sometimes-Never Worksheet on *Polygons*

Directions: Carefully read the following statements about polygons. In the box next to each statement, write an A if the statement is always true, S if it is sometimes true, and N if it is never true. Make sure you explain your reasoning for each statement.

1: A trapezoid is a rectangle.

Reason: _____

2: A quadrilateral is a regular polygon.

Reason: _____

3: Parallelograms are quadrilaterals.

Reason: _____

4: A trapezoid has parallel legs.

Reason: _____

5: The diagonals of a parallelogram bisect each other.

Reason: _____

6: A rectangle is a square.

Reason: _____

7: A square is a rectangle.

Reason: _____

8: A rhombus is a rectangle.

Reason: _____

9: A parallelogram has exactly three right angles.

Reason: _____

10: A rhombus and a rectangle each have four sides of equal length and four angles of equal measure.

Reason: _____

Understanding | Three-Way-Tie



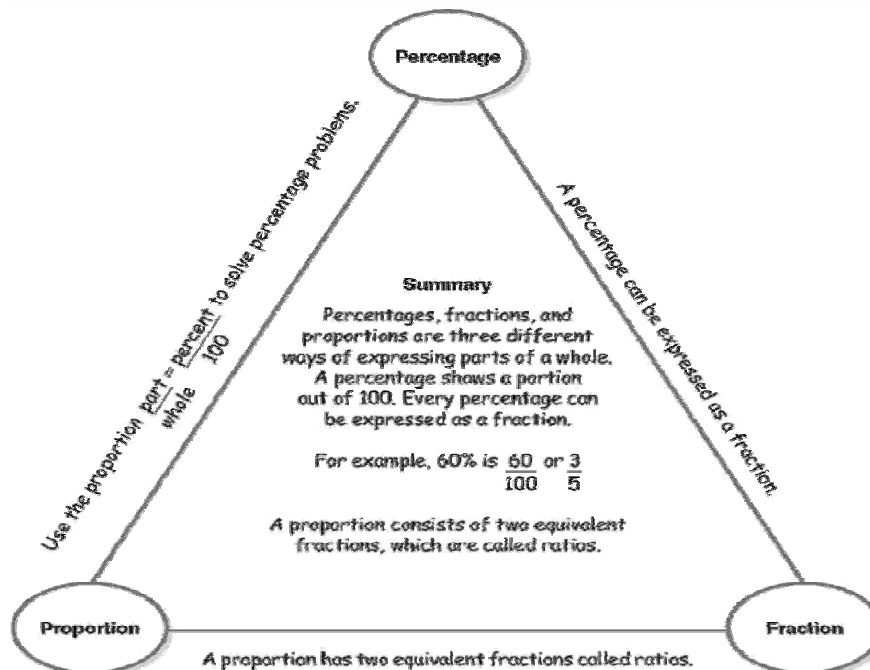
For more about this tool, see pages 108-111 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: A deep understanding of mathematical content means more than knowing what the key concepts are: it also means understanding how these concepts are related, how they fit together to form a bigger picture. Three-Way Tie gives students the opportunity to focus their attention on these hidden relationships. Students identify the relationship between pairs of critical concepts or terms and then distill their understanding of the relationship into a single sentence.

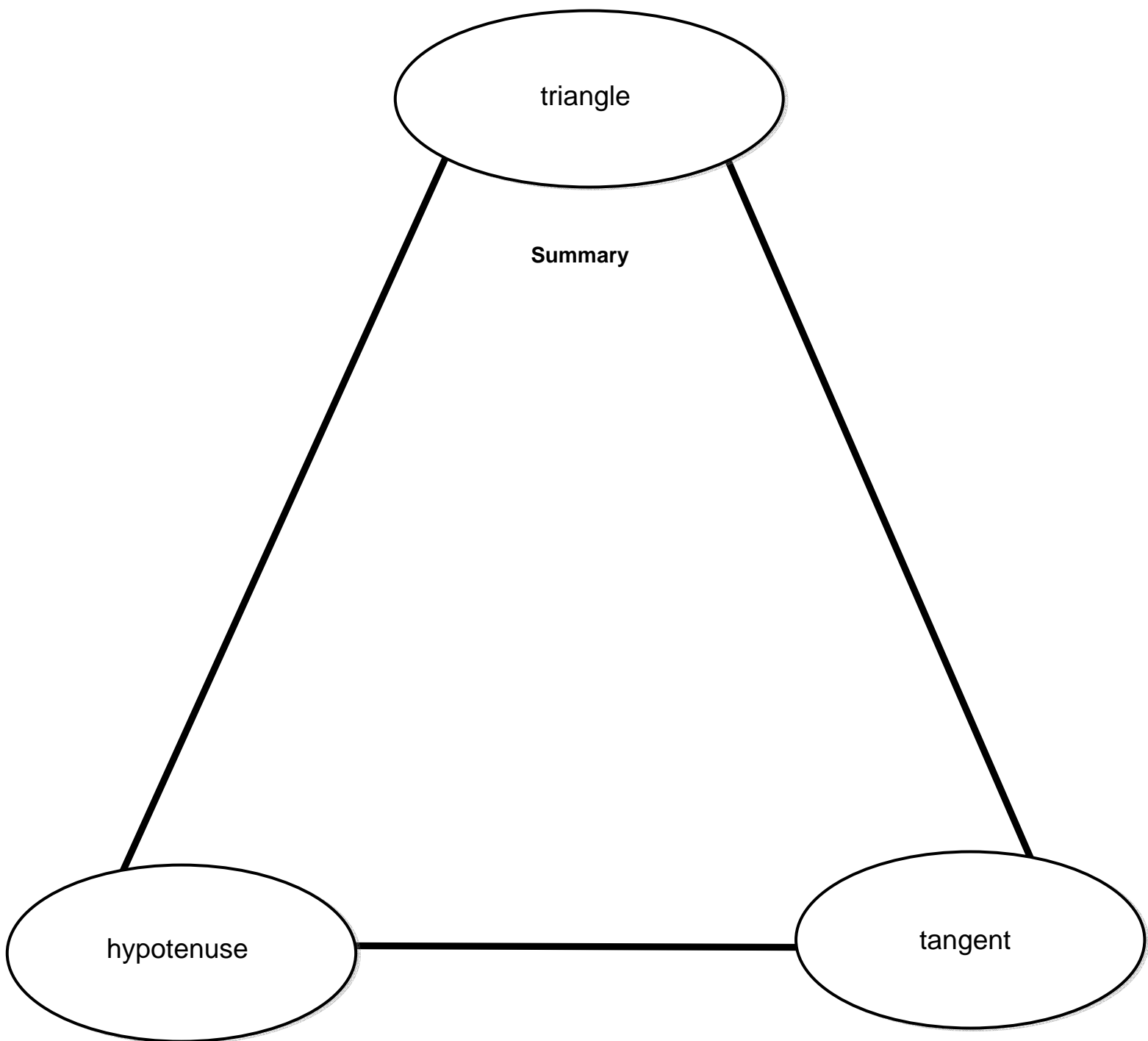
Steps:

1. Identify an important mathematical concept.
2. Graphically “triangulate” the concept with two related terms or concepts. Alternatively, you can have students generate the three terms themselves by selecting the three most important ideas in a reading or unit.
3. Along each side of the triangle, the student writes a sentence that clearly relates the two terms.
4. Have students use their three sentences to develop a brief summary of the concept.
5. Allow students time to share and explain what they wrote on their organizers.

Example:



Name _____ Date _____



Self-Expressive | M + M: Math and Metaphors



For more about this tool, see pages 129-131 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: M + M engages divergent and creative forms of thinking by linking metaphorical thinking and mathematics. Students compare two seemingly unrelated concepts. By finding new and unusual parallels, students deepen their understanding of both the mathematical content and the content they are using to compare against it. The M + M technique taps into the well-known power of metaphors to increase conceptual understanding and academic performance (Cole & McLeod, 1999; Chen, 1999).

Steps:

1. Introduce (or review) the process of metaphorical thinking with your students by comparing two dissimilar concepts or objects. (Often, this is done in the form of a simile: How are parentheses in a mathematics problem like an eggshell?).
2. Review a mathematical concept with students.
3. Allow students to choose a non-mathematical concept or object to serve as a metaphorical counterpart to the mathematical concept, or provide students with a range of choices. Encourage students to explain their metaphors/similes.
4. An alternative to the basic M + M technique is to fill a box or bag up with “stuff”—random items collected from the home or classroom. Students are given a mathematics concept and then pull an item from the bag or box. They then identify as many parallels as they can.

Examples:

- How are fractions like closets?
- How is multiplication like a shortcut through a neighborhood?
- How is long division like following a set of directions to a place you have never gone to before?
- How are number lines like elevators?
- How are pie charts like cakes?
- How are fractions like members of Congress?
- How is factoring like sluicing for gold in a river?
- How are linear equations like people?
- How is the Pythagorean Theorem like the Golden Gate Bridge?
- How are mean, median, and mode in mathematics like RBI, ERA, and OBP in baseball (run(s) batted in, earned-run average, and on-base percentage, respectively)?

M + M: – Math and Metaphors

A metaphor is a literary device used to draw comparisons between two seemingly unrelated concepts. When applied to the study of mathematics, a metaphor becomes a vehicle to deepen “our” understanding of vocabulary and terms, concepts, operations, and other mathematically associated language. For example:

Because . . . *a fraction is cut in half with a numerator on top and a denominator on the bottom.*

And . . . *a pair of scissors cuts things in half, like paper. Try it!*

“A (fraction) is a like a pair of scissors.

_____ is like a _____

Because...

And...

Self-Expressive | Group and Label



For more about this tool, see pages 149-156 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: Group and Label asks students to conceptualize their way to deep understanding by organizing mathematical data into meaningful categories. Students analyze a collection of mathematical information, group the items into categories, and label each category in a way that explains why the items go together. Finally, students use their labeled groups to generate a set of hypotheses or generalizations, which they revisit periodically and refine in light of new learning.

Steps:

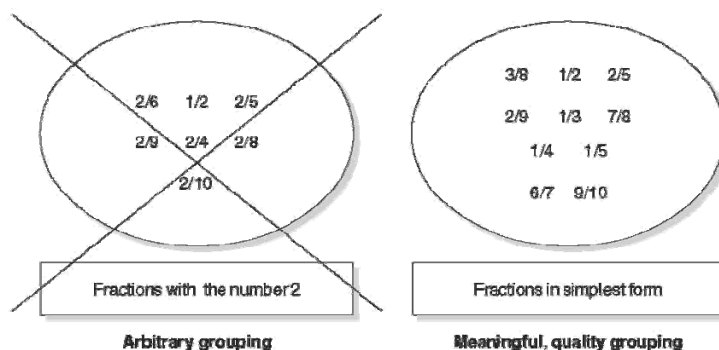
1. Generate a collection of relevant data using terms, numbers, percentages, symbols, etc. Make sure the items are a heterogeneous mixture; students should not be making one or two groups.
2. Model the grouping and labeling process with students. Make sure students understand the difference between groups that are mathematically meaningful and those that are arbitrary.
3. Have students group and label the data either in teams or individually. Be sure to specify group-and-label guidelines; e.g., data can or cannot be used in more than one group.
4. Encourage students to think deeply by considering moves such as combining groups and looking for less obvious connections.
5. Have students use their labeled groups to record a set of predictions on a three-column Prediction Organizer.
6. Facilitate a classroom discussion where students share some of their groups and labels, the reasons behind their groupings, and their predictions.
7. Allow students to revisit, refine, and revise their predictions as they learn more about the content.

Examples:

Fractions

Fractions

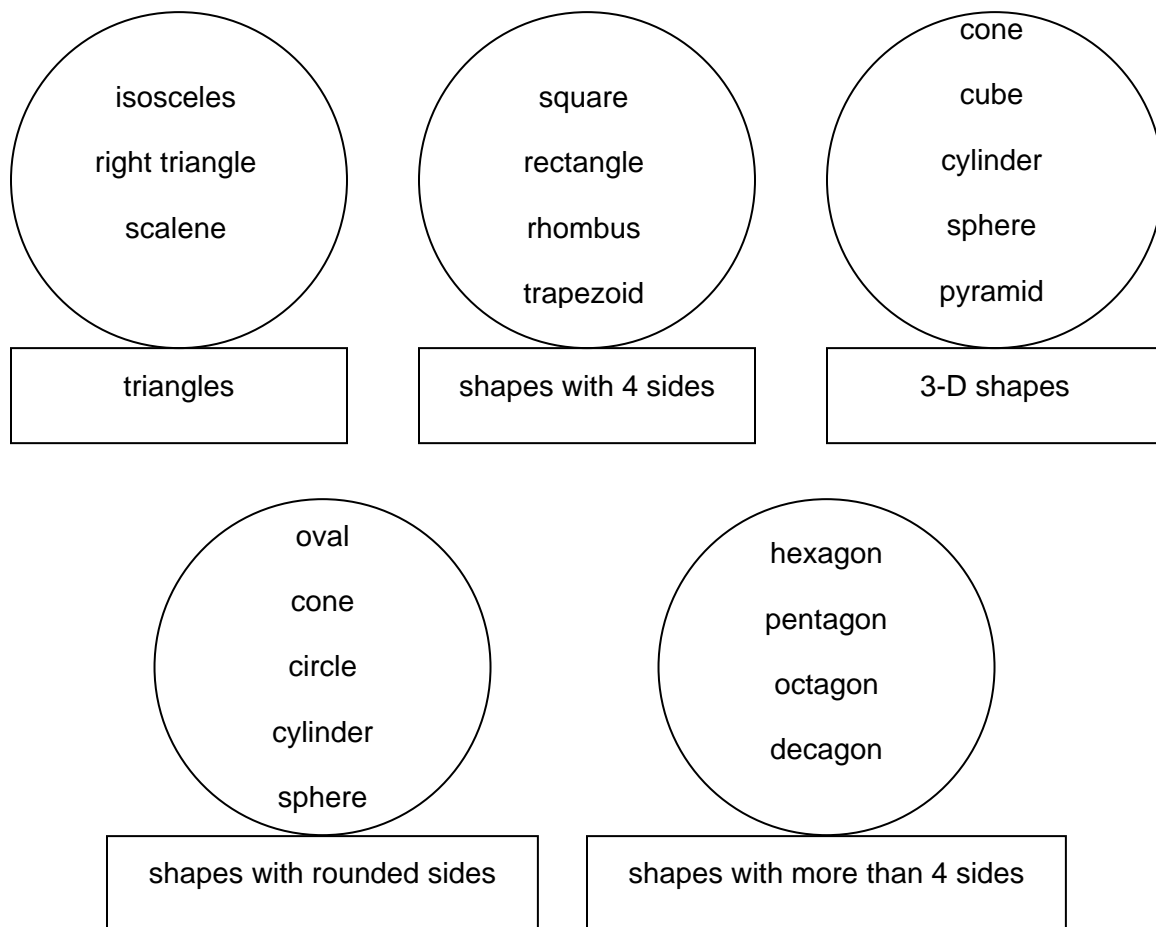
$\frac{3}{8}$ $\frac{2}{6}$ $\frac{1}{2}$ $\frac{2}{5}$
 $\frac{2}{9}$ $\frac{2}{4}$ $\frac{1}{3}$ $\frac{2}{8}$
 $\frac{7}{8}$ $\frac{1}{4}$ $\frac{4}{8}$ $\frac{1}{5}$
 $\frac{2}{10}$ $\frac{5}{10}$ $\frac{6}{7}$ $\frac{9}{10}$



List of Geometric Words

isosceles	octagon	decagon	square	cylinder	oval
rectangle	right triangle	circle	pentagon	rhombus	scalene
hexagon	sphere	cone	pyramid	cube	trapezoid

Students' Groups and Labels



List of Math Words

sum	less than	diminished by	difference	times
reduce by	quotient	more than	take away	triple
multiply	added to	division	double	product
increased by	minus	add	ratio	plus

A set of three overlapping circles arranged horizontally. Below them is a long horizontal rectangle divided into three equal-width columns, each aligned with one of the circles.

A second set of three overlapping circles arranged horizontally, identical to the first set. Below them is another long horizontal rectangle divided into three equal-width columns, each aligned with one of the circles.

Name: _____ Date: _____

Supporting Statements	Predictions	Refuting Statements

Interpersonal | Who's Right?



For more about this tool, see pages 196-198 in *Math Tools, Grades 3-12: 64 Ways to Differentiate Instruction and Increase Student Engagement*.

Purpose: Who's Right? presents students with a scenario in which two people are having difficulty reaching an agreement. At the center of the dispute is a mathematical problem, situation, or claim. By asking students to examine the situation closely and apply mathematical concepts and procedures to determine Who's Right?, the strategy engages students in deep and personal forms of thinking.

Steps:

1. Select or generate a problem with two or more possible solutions. Decide whether the problem will be quantitative (right/wrong answers) or qualitative (analyzing claims and developing a mathematically sound position).
2. Situate the problem inside the context of a dispute between two or more people
3. Have students carefully read (or listen to) the scenario and study whatever positions or decisions were stated.
4. Allow students time to consider the problem and work, individually or in small groups, to determine Who's Right?
5. Facilitate a classroom discussion in which students have the opportunity to explain their choices.

Examples:

Average Autumn Temperature

On the calendar below are written the high and low temperatures (in Fahrenheit) for one week in October. What is the average temperature for the week?

October						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
4	5	6	7	8	9	10
58	60	61	58	57	62	63
49	51	50	48	46	52	55

Three students responded to the question with three different answers. Who is right? Why do you think that student is right?

Alex's answer: 55

Carmen's answer: 56

Gina's answer: 58

Can you explain how each student might have found his or her answer?

(*Note:* The correct answer is 55; average temperature usually is found using the mean of the daily averages of highs and lows).

Pizza Problem

On their way back from a basketball game, four brothers stop at a pizzeria for dinner. The brothers decide to order a large square pizza and share it. Terrence, the oldest brother, says that since the pizza is cut into 9 equal slices, the oldest brother should get three slices and the younger brothers should get two slices each. The second brother, James, says that he knows of a way to cut the square pizza so that each brother has an equal amount of square slices. Not wanting to lose out on any pizza, both younger brothers say James, is right. Who's right—how could the square pizza be cut so that each brother gets an equal amount of square slices.

Bring this completed form to your presenter

Name:	Position/Title:
Organization:	Address:
Work Phone:	Preferred e-mail (please print clearly):

Three ideas from our work today:	One thing I would tell a friend about this workshop:
Before today I thought:	Circle one and explain your choice. Today was more like riding <i>a bike</i> , <i>a walk in the park</i> , <i>a sunrise/sunset</i> , <i>mountain climbing</i> .
Now I think:	